# Self-Organized Synchronization in Decentralized Power Grids

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Robust synchronization (phase locking) of power plants and consumers centrally underlies the stable operation of electric power grids. Despite current attempts to control large-scale networks, even their uncontrolled collective dynamics is not fully understood. Here we analyze conditions enabling self-organized synchronization in oscillator networks that serve as coarse-scale models for power grids, focusing on decentralizing power sources. Intriguingly, we find that whereas more decentralized grids become more sensitive to dynamical perturbations, they simultaneously become more robust to topological failures. Decentralizing power sources may thus facilitate the onset of synchronization in modern power grids.

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The drastic change of electric power supply in the upcoming decades provides an extraordinary challenge for the operation of future power grids [1–5]. For decades, electrical energy was almost exclusively provided by relatively few large power plants mainly based on coal, gas, or nuclear power. As actual political plans indicate, in the near future energy will mostly be provided by a large number of wind parks, photovoltaic arrays, biogas power generators, and other renewable energy sources [1]. One major question is how to ensure stable operation of the entire grid given widely distributed, predominantly small power sources [2].

Stable operation of power grids is based on maintaining a phase-locked, partially synchronous state of the entire system [3,4]. Currently, the synchronization of a power grid is achieved by actively controlling every power generator connected to the network. To cope with the more distributed supply in the future, this strategy shall be extended to controlling the consumer side via a so-called ‘‘smart grid’’ (see, e.g., [5,6]). Exactly how to realize this remains an open challenge, in part because coarse-scale descriptions of self-organized collective dynamics of power grids are scarce.

So far, researchers have considered power grid models from two major classes: (i) abstract, large-scale network models [7–10] for characterizing flow, quasistatic, or probabilistic features of the grid and (ii) detailed, component-level models of engineering used for specific simulations [11]. Whereas the former class is accessible to methods of statistical physics or nonlinear dynamics, it provides only statistical information or a pure dc picture. The latter provide a very detailed picture of power grids, but they demand a huge number of parameters and variables, thereby restricting general insights about dynamics on large scales. How the nonlinear dynamics of complex power networks self-organizes to exhibit synchrony thus remains not fully understood, in particular, for grids that decentralize further.

In this Letter, we contribute towards bridging this gap by investigating the dynamics of nonlinear oscillator networks as power grid models. A bifurcation study shows that normal operation and power outage coexist already in elementary model grids, implying a coexistence regime in all larger networks that contain such elementary ones. We demonstrate that and how power grids with connection topologies as complex as the British transmission grid may collectively phase-lock in a self-organized way and how they may fail. Intriguingly, whereas more decentralized grids tend to be less robust to dynamical perturbations, they simultaneously become more robust against structural perturbations to the grid topology.

*Nonlinear oscillator networks as a power grid model.*— To reveal essential aspects of the oscillatory dynamics of power grids and principal collective phenomena emerging through their nonlinear dynamics, we consider coarse-scale oscillator models of power grids. They are derived from the physics of synchronous generators (representing power plants) and motors (representing consumers), generalizing [12] to complex networks (cf. also [13]). Both types of unit obey the same type of equation of motion with a parameter $P$ giving the generated ($P > 0$) or consumed ($P < 0$) power. The state of each unit $j$ is determined by its mechanical phase $\theta_j(t) = \Omega t + \phi_j(t)$ and phase velocity $\dot{\theta}_j(t)$, where $\Omega = 2\pi \times 50$ Hz (or 60 Hz) is the grid's reference frequency. The equation of motion for the phase deviation $\phi_j(t)$ is obtained via the principle of energy conservation: The generated or consumed power $P_{\mathrm{source},j}$ of each element $j$ must equal the power exchanged with the grid $P_{\mathrm{trans},j}$ plus the accumulated $P_{\mathrm{acc},j} = \frac{1}{2} I \frac{d}{dt} (\dot{\theta}_j)^2$ and the dissipated power $P_{\mathrm{diss},j} = \kappa(\dot{\theta}_j)^2$. Here $\kappa$ is a friction coefficient, and $I$ is the moment of inertia. The power flow between two elements $i$ and $j$ depends on their phase difference and is given by $P_{\max,ij} \sin(\theta_i - \theta_j)$, where $P_{\max,ij}$ denotes the maximum capacity of the transmission line connecting the two elements; cf. [12]. The energy conservation law reads

 

$$P_{\mathrm{source},j} = P_{\mathrm{diss},j} + P_{\mathrm{acc},j} + \sum_i P_{\max,ij} \sin(\theta_i - \theta_j). \quad (1)$$

Assuming $|\dot{\phi}| \ll \Omega$, one finds the equation of motion

$$\frac{d^2\phi_j}{dt^2} = P_j - \alpha \frac{d\phi_j}{dt} - \sum_i K_{ij} \sin(\phi_i - \phi_j), \quad (2)$$

where we abbreviate $P_j = (P_{\mathrm{source},j} - \kappa\Omega^2)/(I\Omega)$, $\alpha = 2\kappa/I$, and $K_{ij} = (P_{\max,ij})/(I\Omega)$. A steady state can exist only if $\sum_j P_j = 0$; i.e., the total consumed ($P_j < 0$) matches the generated power ($P_j > 0$).

*Coexistence in elementary grids.*—Already the most basic grid topology exhibits a regime of coexistence of normal operation and power outage; see Fig. 1. Consider a two-node topology consisting of one generator ($P_g = +P_0$) and one consumer ($P_c = -P_0$). The phase difference $\Delta\phi = \phi_g - \phi_c$ satisfies

$$\frac{d^2\Delta\phi}{dt^2} = 2P_0 - \alpha \frac{d\Delta\phi}{dt} + 2K \sin(\Delta\phi), \quad (3)$$

the equation of motion of damped nonlinear pendulum that is driven by a state-independent force. For $P_0 > K$, no fixed point exists (the dynamics of phase differences approaches a limit cycle) such that stable grid operation is not possible. For $P_0 < K$, there is one attractive fixed point at

$$\Delta\phi = \arcsin(P_0/K) \quad (4)$$

representing stable grid operation. In addition, a limit cycle exists if the increase in kinetic energy due to $P_0$ compensates the decrease due to friction. This is possible only for $P_0 \gtrsim 4\alpha\sqrt{K}/\pi$ assuming weak damping (small $\alpha$); cf. [14]. Figure 1(b) illustrates such coexistence of a fixed point (normal operation) and the limit cycle (power outage). To avoid costly overcapacity of transmission lines, major power grids are often operated in such a region of coexistence where the actual load of a transmission line is of the same order of magnitude as the line capacity (but still below it). In this coexistence regime, however, the collective dynamics depends crucially on the initial conditions, a situation not covered by flow or dc models. Given a coexistence regime for a basic $N = 2$ network, this implies a coexistence regime in any larger network of arbitrary topology as long as it contains that 2-unit system as a subnetwork.

*Synchronization in complex networks.*—Larger networks of complex topologies equally exhibit coexistence and self-organized synchrony [15]: For instance, Fig. 2 shows the dynamics of a network with the coarse-scale topology of the British grid [cf. Fig. 2(a) and [9] for the topology], explicating the possibility of self-organized synchronization. If the capacity of the transmission lines is too small [cf. Fig. 2(b)], no steady state of the power grid exists. The generators accumulate energy such that their phases $\phi_j(t)$ are accelerated while the majority of consumers slow down. Notably, the generators do not desynchron-

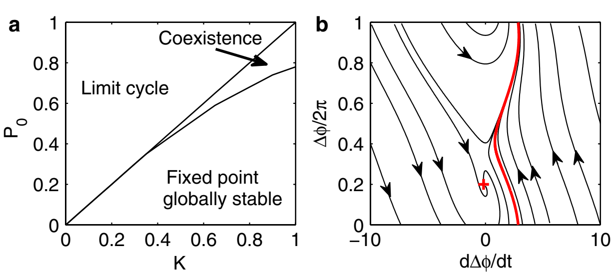


FIG. 1 (color online). Coexistence of normal operation (fixed point) and power outage (limit cycle) in the dynamics of an elementary network (one generator connected to one consumer, $\alpha = 0.1\ \mathrm{s}^{-1}$). (a) Stability phase diagram in parameter space. (b) A stable fixed point (red cross: stable operation with constant power flow) and a limit cycle (red line: no phase locking and fluctuating power flow) coexist ($P_0 = 1\ \mathrm{s}^{-2}$, $K = 1.1\ \mathrm{s}^{-2}$).

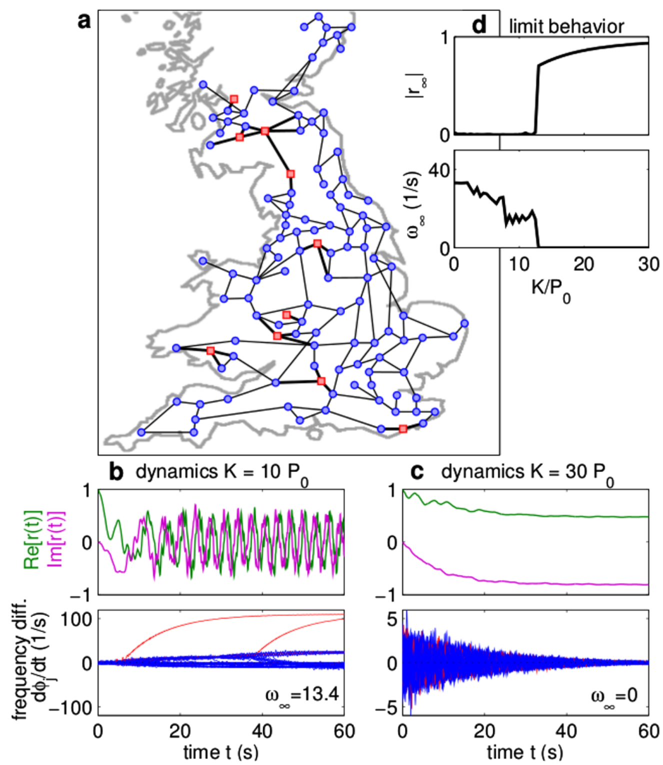


FIG. 2 (color online). Transition to self-organized synchronization in a complex power grid. (a) Topology of the British power grid, consisting of 120 nodes and 165 transmission lines (thin black lines) [9]. Ten nodes are randomly selected to be centralized power plants ($P_j = 11P_0$, □); the others are consumers ($P_j = -P_0$, ○). Power plants are connected to their neighbors with a higher capacity $cK$, $c \geq 1$ (thick lines); the remaining transmission lines have a capacity $K$. (b), (c) Dynamics of the generators' (red) and consumers' (blue) frequencies $d\phi_j/dt$ and the respective order parameter $r(t)$ for (b) weak and (c) strong coupling. (d) The order parameter $r_\infty$ and the asymptotic mean frequency difference $\omega_\infty$ as a function of the coupling strength $K$ for $c = 2$.

ize at once but rather in a cascade of failures (cf. also [7,9]). Because of the damping, the system tends towards a limit cycle where the average frequency difference $\omega = \sum_j |d\phi_j/dt|/N$ assumes a nonzero value $\omega_\infty > 0$. If the transmission lines are strong enough, all units of the power grid synchronize [Fig. 2(c)], stable operation is possible without an active phase control of the units, and $\omega_\infty = 0$. The phase ordering of the power grid is measured by the order parameter $r(t) = \sum_j e^{i\phi_j(t)}/N$, respectively its long time average

$$r_\infty := \lim_{t_1\to\infty} \frac{1}{t_2} \int_{t_1}^{t_1+t_2} r(t)dt, \qquad (5)$$

assuming a sufficiently large time interval $t_2$. Both the average frequency $\omega_\infty$ and the order parameter $r_\infty$ are plotted as a function of the coupling strength $K$ in Fig. 2(d), showing the onset of synchronization above a critical coupling strength $K \geq K_c \approx 13P_0$.

Thus, the power grid model presented here captures essential features of a real grid, including self-organized synchrony and its coexistence with power outage.

*Dynamic stability.*—How does decentralization impact the system's stability to dynamic perturbations? In the model, we successively replace large power plants ($P_j = +11P_0$) by smaller ones ($P_j = +1.1P_0$, ten per large plant). We test the stability against fluctuations on the consumers' side by transiently increasing (away from stationarity) the power demand of each consumer during a short time interval (here, 5 s). The condition of equal $\sum_j P_j$ on both the consumers' and producers' side is violated during the perturbation, and therefore the system cannot remain in a stable state.

After the perturbation is switched off, the system either relaxes back to a steady state or does not, depending on the strength of the perturbation, as illustrated in Figs. 3(a) and 3(b), respectively. The results are summarized in Figs. 3(c) and 3(d). We find that the maximally allowed perturbation strength shrinks with decentralization, but still all grids are stable up to strengths a few times larger than the unperturbed load [Figs. 3(c) and 3(d)].

*Decentralization supports synchrony.*—It has been questioned whether a network of many small, distributed power sources can be effectively synchronized without the help of a reference signal generated by large power grids (see, e.g., [3]). We find that self-organized synchronization is even promoted when more but smaller and decentralized sources are present. Figures 4(a)–4(c) show how the synchrony of the power grid is affected by this procedure. Most interestingly, the phase order parameter $r_\infty$ *increases* with decentralization. At the same time, the average frequency $\omega_\infty$ decreases. The critical coupling strength $K_c$ for the onset of synchronization thus decreases; i.e., synchrony can be realized already with less transmission capacity.

Intuitively, the transmission lines connecting the power plants to the rest of the grid are heavily loaded and thus most likely to fail. Stability thus would be increased just because these lines become less loaded if large generators are replaced by (several) smaller ones. This notwithstanding, decentralization itself, by its more distributed nature, supports synchrony, not sensitively depending on local line overload. To show this, we increased line capacities connecting the power plants to the grid compared to other lines by a factor $c$. Self-organized synchronization is still promoted by decentralization for $c = 2$ [Fig. 4(b)] and even for $c = 10$, completely compensating the tenfold difference in plant power [Figs. 4(c) and 4(d)]. Further studies on small-world and other grid topologies show qualitatively the same results. We conclude that, at least for stationary operation, further decentralizing a grid promotes self-organized synchronization.

*Decentralization supports robustness against structural damage.*—The grid is also robust against structural damages of its interconnections. We have simulated the impact

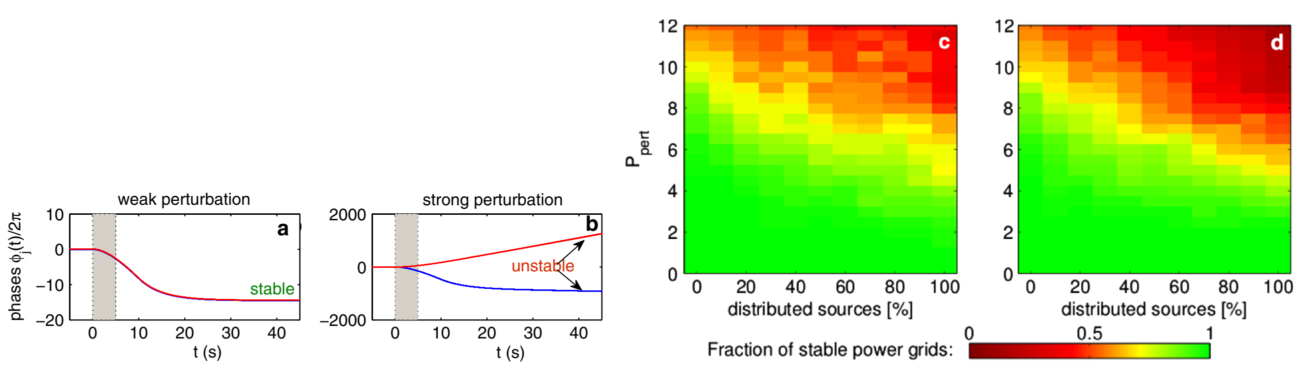


FIG. 3 (color online). Stability of a power grid against perturbations in power demand. (a),(b) Time evolution of the phases of the generators (red) and consumers (blue) in case of a weak (a) and strong (b) perturbation in the time interval 0–5 s (shaded area). (c),(d) The color maps indicate the fraction of random grids which are unstable against a perturbation as a function of the perturbation strength $P_{\text{pert}}$ and the fraction of small distributed generators. Numerical results have been averaged over 100 realizations, where the replacing smaller power sources were randomly placed in the grid. Parameters are $K = 20$ and $c = 2$ in (c) and $K = 20$ and $c = 10$ in (d).

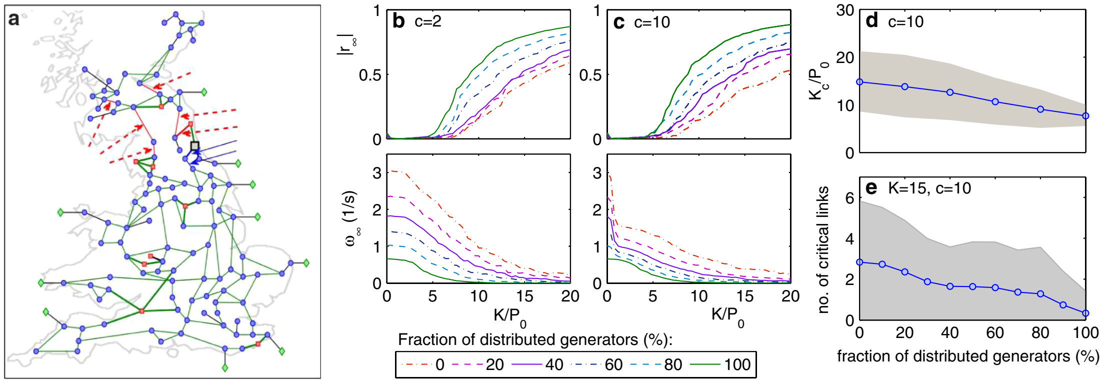


FIG. 4 (color online). Decentralizing power plants decreases synchronization threshold and increases robustness to structural damage. This figure illustrates simulated future development of the British grid, replacing large centralized power plants by small distributed ones. (a) First step of decentralizing: One power plant (marked by black square) is disabled; i.e., the generated power is set to zero ($P_j = 0$). Instead, ten new small generators are added to the grid at random positions (green diamonds). The color of the links illustrates their relevance for the global structural stability of the power grid (see the main text). Removing certain single links causes a power outage: Two links (blue solid arrows) are crucial for the stability of the initial grid, but not anymore when the marked power plant is replaced by distributed generators. Six links (red dashed arrows) are crucial in both cases. Parameters are $K = 12$ and $c = 10$. To increasingly decentralize the grid, randomly chosen single large plants are successively replaced by ten smaller ones. (b), (c) Promotion of self-organized synchronization due to the replacement of centralized power plants. Panel shows order parameter $|r_\infty|$ and mean frequency $\omega_\infty$ as a function of coupling strength $K$ for $c = 2$ (b) and $c = 10$ (c), respectively. (d) Change of critical coupling strength $K_c$ for the onset of synchronization due to the replacement of the centralized power plant for $c = 10$. (e) Change of structural stability. Panel shows a number of critical links in the network, whose removal leads to a loss of synchrony and thus a major power outage, discarding bridges. Here, coupling strength is fixed to $K = 15$ and $c = 10$. Quantities in (b)–(e) are averaged over 100 realizations, as in Fig. 3. The shaded areas in (d) and (e) show standard deviation.

of the damage of single transmission lines on the stability of the power grid. An example is shown in Fig. 4(a) for the British power grid, comparing the stability properties of a fully centralized grid with one where 10% of the plants are decentralized. The importance of each link for the stability of the synchronized state is indicated by the color of the lines. Green indicates noncrucial lines that can be removed without losing synchrony for both grids. Black indicates bridges, whose removal disconnects the grid. Two transmission lines (blue solid arrows) are crucial for the stability of the initial grid, but not anymore when the marked power plant is replaced by distributed generators. Six lines (dashed red arrows) are crucial in both cases.

Moreover, for power grids with smaller, more distributed generators, the number of crucial links decreases such that a global failure is less likely [Fig. 4(e)]. Further inspection of the detailed consequences of removing links suggests a rough intuitive explanation: The probability of a global failure is highest when in the immediate neighborhood there is no pathway which can take over the respective power load. This is more often the case for few, large power plants, as there are less transmission paths in the network. We conclude that replacing large power plants by distributed generators may not only promote synchrony but also increase the robustness of the power grid with respect to structural damages.

*Conclusion and outlook.*—In summary, we have analyzed a coarse-scale oscillator model of power grids derived from the properties of the underlying physical machines in the limit of weak damping, generalizing [12] to complex networks. The model exhibits collective synchrony and coexistence of stable operation and power outage, as in real grids. Still, the model class is simple enough to reveal basic principles, simulate larger-scale networks, and understand collective dynamic phenomena on complex topologies.

Intriguingly, we found two counteracting tendencies due to decentralization: As might be expected, networks become less stable against short-term, large-amplitude dynamic perturbations with increasing decentralization. At the same time, networks with more distributed power sources are less vulnerable to transmission line failures, i.e., structural perturbations. This suggests that real-world grids with a large fraction of renewable energy sources will require control as anticipated. Yet it seems that such control will be required due to the ongoing dynamic fluctuations only and not caused by decentralization that in turn harnesses inhomogeneities and fluctuations given an extended regime of stable synchrony.

Taken together, our results indicate that decentralizing power sources may moderately decrease the grids' dynamic stability, but support the onset of self-organized synchrony, and make it more robust to structural damages. It is of great scientific and economic interest to understand how synchronization depends on details of the topology of power grids and to derive viable strategies for how to establish new transmission lines. For instance, an enormous challenge for the development of power grids is that often renewable energy sources are built predominantly at the seaside such that energy is generated far away from consumers. The insights and methods presented here may further our understanding of the collective dynamics of today's power grids as well as help investigating different scenarios for upgrading the grid.

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[15] From now on, we assume that a basic consumer (representing, e.g., a small town) receives power of 105 MW and $I \approx 2 \times 10^4$ kg m$^2$, $\kappa \approx 10^3$ kg m$^2$/s. We set $P_0 = 1$ s$^{-2}$ and the damping constant $\alpha = 0.1$ s$^{-1}$.